$$\int \frac{1}{x}\,dx = \ln x + C$$

$$\cancel{y = -x}$$

$$y = \ln x$$

$$\frac{dy}{dx} = \frac{1}{x}$$

$$\int \frac{1}{2x+1}\,dx = \int \frac{1}{u} \cdot \frac{du}{2}$$

$$u = 2x+1$$

$$du = 2\,dx$$

$$\frac{du}{2} = dx$$

$$= \frac{1}{2}\int \frac{1}{u}\,du$$

$$= \frac{1}{2}\ln|u| + C$$

$$y = \frac{1}{2}\ln|\underline{2x+1}| + C$$

Check:

$$\frac{dy}{dx} = \frac{1}{\cancel{2}} \cdot \frac{1}{(2x+1)} \cdot \cancel{2}$$

$$\int e^{x}\,dx = e^{x} + C$$

$$\int e^{x+1}\,dx = \int e^{u}\,du = e^{u} + C$$

$$y = e^{x+1} + C$$

$u = x+1$

$du = dx$

$$\int e^{2x}\,dx = \int e^{u} \cdot \frac{du}{2}$$

$u = 2x$

$$= \frac{1}{2}\int e^{u}\,du$$

$du = 2dx$

$\frac{du}{2} = dx$

$$\frac{1}{2}e^{u} + C$$

$$y = \frac{1}{2}e^{2x} + C$$

Check:

$$\frac{dy}{dx} = \frac{1}{\cancel{2}} \cdot e^{2x} \cdot \cancel{2} = e^{2x}$$

$$\int \frac{x+3}{x^2+6x+7}\,dx = \int \frac{1}{u}\cdot\frac{du}{2}$$

$$= \frac{1}{2}\int \frac{1}{u}\,du$$

$u = x^2+6x+7$

$du = (2x+6)\,dx$

$\frac{du}{2} = (x+3)\,dx$

$$= \frac{1}{2}\ln|u| + C$$

$$y = \frac{1}{2}\ln\left|x^2+6x+7\right| + C$$

Check:

$$\frac{dy}{dx} = \frac{1}{\cancel{2}}\cdot\frac{1}{x^2+6x+7}\cdot\overset{x+3}{(\cancel{2}x+\cancel{6})}$$

$$= \frac{x+3}{x^2+6x+7}$$

$\dashv$

$$\int \frac{e^{3/x}}{x^2}\,dx = \int e^u \cdot \frac{du}{-3}$$

$$\cancel{u = x^2}$$

$$\cancel{du = 2x\,dx}$$

$$u = \frac{3}{x} = 3x^{-1}$$

$$du = -3x^{-2}\,dx$$

$$\frac{du}{-3} = x^{-2}\,dx$$

$$\frac{du}{-3} = \frac{1}{x^2}\,dx$$

$$= -\frac{1}{3}\int e^u\,du = -\frac{1}{3}e^u + C$$

$$y = -\frac{1}{3}e^{3/x} + C$$

Check:

$$\frac{dy}{dx} = -\frac{1}{3}\cdot e^{3/x}\cdot -3x^{-2}$$

$$\frac{3}{x} = 3x^{-1}$$

$$\frac{dy}{dx} = \frac{e^{3/x}}{x^2}$$

$$\int \boxed{x^2} e^{x^3} \boxed{dx} = \int e^u \cdot \frac{du}{3}$$

$$u = x^3$$

$$du = 3x^2 dx$$

$$\frac{du}{3} = x^2 dx$$

$$= \frac{1}{3} \int e^u \, du = \frac{1}{3} e^u + c$$

$$y = \frac{1}{3} e^{x^3} + c$$

5. $\int \frac{1}{2} t \cos 4t^2 \, dt = \frac{1}{2} \int \cos u \, \frac{du}{8}$

$u = 4t^2$

$du = 8t\,dt$

$\frac{du}{8} = t\,dt$

$= \frac{1}{16} \int \cos u \, du$

$= \frac{1}{16} \sin u + C$

$y = \frac{1}{16} \sin 4t^2 + C$